19/10/19

of bending. Pg-1

Diffraction :- The phenomenon of light around an obstacle and its spreading into the area of geometrical shadow is called as Diffraction of light.

The size of obstacle must be comparable to wavelength of light for diffraction phenomenon.

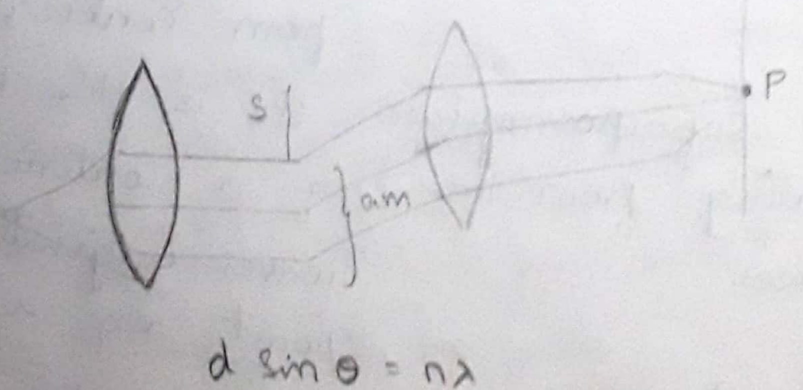
There are two types of diffraction.

i) Fraunhoffer Difraction.

ii) Fresnel Diffraction.

i) Franhoffer Diffraction :-

* Source and the screen are at infinite distance (parallel wave front).
* Incident wavefronts on diffracting obstacle are plane.
* Diffracting wavefronts are converged by means of a convex lens to produce diffraction pattern [Easy to mathematically handle].



$$d \sin\theta = n\lambda$$

$$\frac{a}{m} \sin\theta = n\lambda$$

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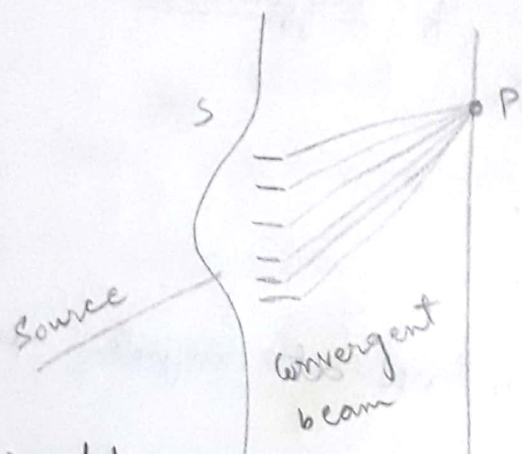
iii) Frensel Diffraction.

* Source and screen are at a finite distance from each other.

* Incident wavefronts are either spherical or cylindrical

* Convex lens is not needed to converge the spherical wavefront.

[Easy to experimentally setup.]

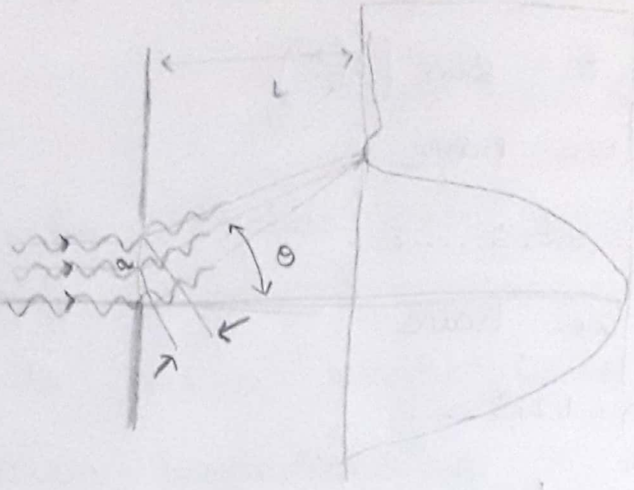


Different b/w Interference & Diffraction.

Interference	Diffraction.
It is has a number of equally spaced bright and dark bands.	It has central bright maximum which is twice as wide as the other maxima. [The intensity falls as we go to successive maxima away from centre; on either side.
It is due to superposing two waves originating from the coherent sources.	It is due to the superposition of a continous two coherent waves originating from different point on a single source.
Maxima $\frac{xd}{D} = n\lambda$ $n = 0, 1, 2, 3 \ldots$	Minima $d \sin\theta = n\lambda$ $n \neq 0$ $n = 1, 2, 3 \ldots$
Minima $\frac{xd}{D} = \left[n + \frac{1}{2}\right]\lambda$ $n = 0, 1, 2, 3 \ldots$	Maxima $d \sin\theta = \left[n + \frac{1}{2}\right]\lambda$ $n = 1, 2, 3 \ldots$

Single slit Diffraction

Consider a slit of width a as shown



single slit

i)

* When monochromatic light of wavelength λ is incident the light arriving at a point P on the screen is formed by a superposition of light waves arising from sources on the wave front.

* The condition for Constructive Interference is given by

$$OPD = n\lambda$$

Similarly for dark point.

$$OPD = (n+1/2)\lambda.$$

If we divide the wavefront into two parts
The OPD between the 2 parts is given by

$$\frac{(a \sin\theta)}{2} = n\lambda \quad \text{for bright point.}$$

Similarly for dividing into m parts.

$$\frac{a \sin\theta}{m} = n\lambda$$

$$\sin\theta = \frac{nm\lambda}{a}$$

$$\sin\theta = \frac{n'\lambda}{a} \qquad (n' = mn)$$

$$\theta = \sin^{-1}\left(\frac{n'\lambda}{a}\right)$$

For maximum we have

where $n = 1, 2, 3 \ldots\ldots$

For minimum we have

where $n = 1, 2, 3 \ldots\ldots$

INTENSITY EQUATION / DISTRIBUTION.

We divide the slit into narrow strips with width of Δy. Each strips acts as a source of coherent radiation and the E value from its represented as phaser. Phasers have a length proportional to magnitude & a direction which represent its phase.

Total field at a point P is given by the phasor sum of the phasers $\bar{E}_1 + \bar{E}_2 + \bar{E}_3$ and so on.

The resultant phaser E_0 is found by joining the tail of 1st phaser to the head of the last.

A phase difference of 2π corresponds to a wavelength difference of λ.

Therefore a phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta y \sin\theta$$

... total phase difference between the top and bottom of the slit is given by

$$n\Delta\phi = n\frac{2\pi}{\lambda}\Delta y \sin\theta.$$

$$\phi = \frac{2\pi}{\lambda} a \sin\theta$$

let $\bar{E}_1$ be the Electric field from the first bit of the slit Δy_1 and so on.

The phase $\bar{E}_1$ has a length Equal to the magnitude E_1 and direction Corresponding to its phase.

let $\bar{E}_m$ be the length of the arc.

E_θ be the length of the chord. of radius R subtending angle ϕ.

$$\sin\phi/2 = \frac{E_\theta}{2R}$$

$$E_\theta = 2R\sin\phi/2$$

$$E_\theta = \frac{2E_m}{\phi}\sin\phi/2$$

$$\phi/2 = \alpha, \quad E_\theta = E_m\frac{\sin\alpha}{\alpha}$$

$$I \propto E^2, \quad \frac{I_\theta}{I_m} \propto \left(\frac{E_\theta}{E_m}\right)^2 \propto \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\phi = \frac{2\pi}{\lambda} a\sin\theta$$

$$\alpha = \phi/2 = \frac{\pi}{\lambda} a \sin\theta .$$

$$I \propto \left(\frac{\sin \alpha}{\alpha/\lambda} \right)^2$$

I becomes very small because $\pi \sin\theta$ = Constant

If a is much larger than λ.

Intensity due to Interference.

Consider Youngs Double Slit Experiment and let E_1 be the Electric vector form slit S_1 and E_2 be the vector from slit S_2.

The resultant Electric vector is

$$E_1 = E_0 \sin \omega t , \quad E_2 = E_0 \sin(\omega t + \phi)$$

$$E = E_1 + E_2$$

we know

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) . \cos\left(\frac{A-B}{2}\right)$$

$$\frac{\omega t - \phi/2}{2}$$

$$E = 2 \sin(\omega t + \phi/2) \cos \phi/2$$

let $2E_0 = E_{max}$

let $E_0 = E_{max} \sin(\omega t + \phi/2)$

DIFFRACTION DUE TO DOUBLE SLIT.

Consider a Youngs Double slit Experiment with the width of a slits a and the space between the slits d.

In this Case,

the diffraction in Each of the slits and Interference between the slits.

Thus the resultant distribution is the product of the Effect of diffraction and interference.

$$I = I_{interference} * I_{diffraction}.$$

$$I = I_m \cos^2 \beta \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where,

$\alpha = (\pi * a \sin \theta)/\lambda$

$\beta = (\pi * d \sin \theta)/\lambda$

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Diffraction Grating.

Gratings are made by rulings Equally Spaced parallel grooves on a thin film Coating on a glass plate using a diamond Cutting point.

The no of lines per unit length is denoted by N.

The Grating Element $d = 1/N$ is the spacing b/w adjacent slits.

The intensity pattern is a Series of interference fringes with an angular separation $= \lambda/d$

Relative intensity $= \lambda/a$

In General for a minima; $\Delta\phi = 2\pi/N$

But for interference, $\Delta\phi = (2\pi d \sin\theta)/\lambda$

If θ is Small then $\sin\theta \approx \theta$

$$\theta = \lambda/(N * d)$$

Intensity distribution of a diffraction grafting with N lines per Unit length is given by.

$$I = \left(\frac{A \sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\beta}\right)^2$$

The factor $\left[\frac{A\sin\alpha}{\alpha}\right]^2$ gives the intensity distribution due to single slit while $\left(\frac{\sin N\beta}{\beta}\right)^2$ gives the intensity distribution due to the Combined Effect of all the slits.

Central maximum occurs when in the above Eq^n.

$\alpha \to 0$ and $\beta \to 0$

Then $I = A^2 N^2$

The principle maxima occurs when $\sin\beta = 0, \beta = \pm n\pi$

where $n = 1, 2, 3 \ldots\ldots$

At the same time $\sin N\beta = 0$ and can be found

using

$$\lim_{\beta = \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \pm N$$

The resultant intensity is.

$$I = \left[\frac{A \sin \alpha}{\alpha} \right]^2 N^2$$

The maxima is obtained for $\beta = \pm n\pi$ or

$$(2\pi/\lambda)(a+d)\sin\theta = \pm n\pi$$

$$(a+d)\sin\theta = \pm n\pi$$

where $n = 0$ corresponds to zero order maximum and for $n = 1, 2, 3$ we get the first, second, third order principal maxima respectively.

The $\pm$ sign show that there are two principal maxima of the same order lying on both sides of the zero order maximum.

The minima occur when

$$\sin N\beta = 0$$

But $\sin \beta \neq 0$

For minima $\sin N\beta = 0$

and $N\beta = \pm n\pi$

$$N(2\pi/\lambda)(a+d) = \pm n\pi$$

$$N(a+d)\sin\theta = \pm n\pi$$

when n has all integral value Except 0, N, 2N,

Hence $(N-1)$ secondary maxima between two adjacent principal maxima.

There are $(N-1)$ minima between 2 adjacent principal maxima and there are $(N-2)$ other maxima between two principal maxima.

These are maxima
↓
secondary.